Microeconomics III
Autumn 2020

Re-take Exam Solutions

1. Consider the following normal form game:

		Player 2		
		a	b	c
	A	$1,3$	$0,2$	$4,4$
Player 1	B	$1,2$	$5,5$	$1,3$
	C	$2,1$	$6,0$	$2,0$

(a) (2 points) Which strategies can be eliminated by iterated elimination of strictly dominated strategies? Explain briefly each step (1 sentence).

Solution.
Round 1: B is strictly dominated by C.
Round 2: After eliminating B, a strictly dominates b.
No other actions can be eliminated.

Solution: Pure actions A, C, a, c survive IESDS. (Hence, it is also the unique Nash equilibrium.)

(b) (4 points) Find all (pure and mixed) Nash equilibria of the game. Remember to argue that there cannot be any other Nash equilibria. Calculate the expected payoffs for each equilibrium.

Solution. It is enough to consider mixed (and pure) actions with supports on $\{A, C\}$ and $\{a, c\}$ because B and b are eliminated and hence cannot be part of a NE. First observe that (A, c) and (C, a) are pure strategy Nash equilibria. The corresponding equilibrium payoffs are $(4, 4)$ and $(2, 1)$.

Mixed NE? There cannot be a mixed equilibrium where only one player mixes because neither player gets the same payoff from two actions when the other player plays a pure strategy. The only option left is that both players mix between the first and the last action: $((p, 0, (1-p); (q, 0, 1-q))$. Let's check indifference:

$$1q + 4(1-q) = 2q + 2(1-q) \iff q = \frac{2}{3},$$
$$3p + 1(1-p) = 4p + 0(1-p) \iff p = \frac{1}{2}.$$

Equilibrium payoffs: $(2, 2)$

Solution: NE$= \{(A, c), (C, a), ((1/2, 0, 1/2; (2/3, 0, 1/3))\}$ with payoffs $\{(4, 4), (2, 1), (2, 2)\}$.

(c) (4 points) Now, consider the situation where the game is played twice and players observe each other's period 1 actions before period 2. No discounting. Construct a subgame perfect Nash equilibrium, which Pareto dominates all equilibria where players play a stage Nash equilibrium in both periods. Remember to define full equilibrium strategies.

Solution. Action profile (B, b) would give a higher payoff for both players than any stage NE. Let's try to support (B, b) in the first period as a SPNE.

We can use (A,c) as the reward in the second period and (C,a) (or the mixed NE) as a punishment. Suggested equilibrium strategies are then the following. In the first period, players play (B,b) and in the second period, they follow:

$$\sigma_1 = \begin{cases} A & \text{if } (B,b) \text{ in the first period} \\ C & \text{otherwise} \end{cases}$$

$$\sigma_2 = \begin{cases} c & \text{if } (B,b) \text{ in the first period} \\ a & \text{otherwise} \end{cases}$$

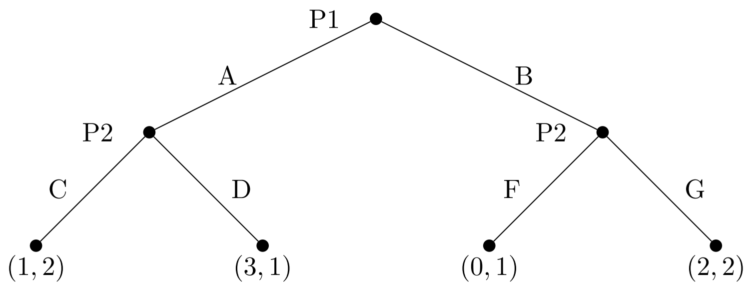
The suggested profile yields payoffs $5+4=9$ for both players, which is more than in any SPNE where a stage NE is played in both periods (max payoff $4+4$).

Is the suggested strategy profile a SPNE? Neither player wants to deviate in the second period because in both cases they play a stage NE and hence best respond to each other. Does Player 1 want to deviate in the first period? The best possible deviation for P1 is to play C instead of B. In that case, P1 gets $6+2$, which is less than what he gets if he follows the suggested strategy. Player 2 plays a static best response in both periods and hence does not want to deviate.

Solution: $((B,\sigma_1);(b,\sigma_2))$ is a SPNE and yields 9 for both players.

2. For each statement below, state whether it is TRUE or FALSE and briefly motivate your answer. Informal discussion is enough (2-5 sentences each).

(a) (4 points) (A,C) is a subgame perfect equilibrium outcome of the following extensive form game:



Solution. FALSE. G is the optimal action for P2 in the second node. Hence, A is not optimal for P1. Backward induction gives a unique SPNE: (B, CG).

(b) (3 points) In Stackelberg duopoly, there is a last-mover advantage because the last mover gets to observe the first-mover's action and hence has more information.

Solution. FALSE. There is a first-mover advantage in Stackelberg duopoly. The first-mover is better off because he can influence the other firm's production decision by producing more.

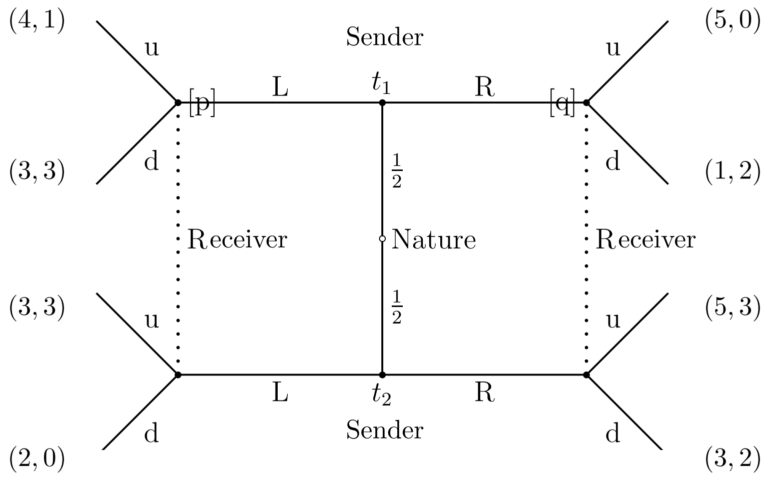
(c) (3 points) Consider a second-price sealed-bid auction with $n > 2$ bidders and independent private values. There always exists a Bayes Nash equilibrium where bidders bid their value.

Solution. TRUE. Independent of what other bidders bid, it is a weakly dominant strategy to bid one's own valuation. In a second-price auction one's own bid does not affect the price when the bidder wins but only whether he wins or not. By bidding the valuation, the bidder makes sure that whenever he wins the price is at most the valuation and whenever he loses the price is above the valuation.

3. Consider the following signaling game:

(a) (3 points) Can pooling on L be the outcome of a perfect Bayesian equilibrium? Argue why/ why not. If yes, find the equilibrium.

Solution. SR3: $p = 1/2$, q is off the equilibrium and can be anything.



SR2R: BR if L: u $(p1 + (1-p)3 > p3 + (1-p)0)$.
BR if R: depends on q.

SR2S: BR if t_1: L if and only if P2 plays d after R.
BR if t_2: L if and only if P2 plays d after R.

SR2R (again): P2 plays d after R if and only if the following holds:

$$q0 + (1-q)3 \leq q2 + (1-q)2 \iff q \geq \frac{1}{3}.$$

Solution: The PBE is $(LL, ud); q \geq 3/5$.

(b) (3 points) Can separating such that type 1 plays L and type 2 plays R be the outcome of a perfect Bayesian equilibrium? Argue why/ why not. If yes, find the equilibrium.

Solution. SR3: $p = 1$, $q = 0$.

SR2R: BR if L: d.
BR if R: u.

SR2S: BR if t_1: R $(3 < 5)$.
The sender's incentive compatibility is not satisfied.

Solution: separating such that type 1 plays L and type 2 plays R cannot be a PBE.

(c) (4 points) Choose one of the following: i) come up with a real life situation that can be modeled as the previous signaling game. Interpret your results in (a) and (b) from the perspective of the chosen application (2-4 sentences). OR ii) Choose any application where signaling can be relevant and write it as a signaling game. Formally or informally, analyze your chosen game and briefly discuss. If you choose alternative ii), you are not allowed to use any example from the lectures, assignments, or problem sets.

Solution. i) One example: Sender is an employee who chooses between 'specialist' track (L) and 'management' track (L) in the firm. The employee can be either strong with details (t_1) or a good leader (t_2). The employer (Receiver) either gives a promotion to the employer (action u) or not (action d). The employer is better off if she promotes a good leader. Both types would like to get promoted but their payoffs differ: the detail type's worst outcome is to be in the management track without a promotion, while being in the specialist track without a promotion is the worst for the leader type.

Given the payoff structure, the leader type cannot separate from the detail type by choosing the management track because then the detail type would want to mimic him in order to get promoted there (see part (b)). Instead, both types may go to the specialist track. In that case, the employer promotes both types because she thinks that it is likely enough that the employee's type is leader (part (a)).